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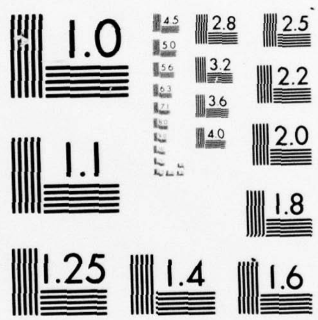

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Interim Scientific Report for 1977

OPTIMIZATION PROBLEMS:

DUALITY AND MULTIPLIER METHODS

Grant AF-AFOSR-77-3204

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This describes five artices, a doctoral dissertation and part of a book manuscript produced under the grant in question in 1977.

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During this first year under Grant AF-AFOSR077-3204, five articles [1], [2], [3], [4], [5] were produced by the Principal Investigator, R. T. Rockafellar. Writing of the book manuscript [7] continued. A doctoral thesis [6] was written by J. W. Spingarn.

Network optimization. Major effort during the summer of 1977 went into the book, "Optimization in Networks," [7], which however is still not complete. Three new chapters were written, totaling over 200 typed pages. The contents of these chapters are listed at the end of this report. Much of this portion is concerned with duality-based algorithms for determining optimal flows and potentials, and some of these algorithms are completely new. There are numerous applications to new problems involving transportation networks, logistics, traffic routing, stockpiling, pipe systems, force distribution in structures, and other situations in operations research that over the years have ingeniously been represented in terms of network flows and potentials.

Stochastic programming. Continued collaboration with R. J. B. Wets has resulted in paper [1], "The optimal recourse problem in discrete time: L^1-multipliers for inequality constraints." We feel this is the high point so far in the series.

An optimal recourse problem is an optimization problem with both stochastic and dynamic aspects, involving the interplay of observations and responses. In discrete time (with a finite horizon), there are finitely many stages, at each of which a decision is selected on the basis of prior observations of random events and subject to costs and constraints affected by these observations as well as by past decisions. The goal is to minimize expected cost, taking into account the known distribution of future random events. Paper [1] is concerned with the derivation of necessary and sufficient conditions for optimality in the case of convex costs and constraints.

It is shown that if the recourse problem is "strictly feasible and satisfies a new condition called 'essentially complete recourse,'" optimal solutions can be characterized by a pointwise Kuhn-Tucker

property involving only L^1-multipliers (no singular components!). Applications to multistage stochastic programs with special structures are developed in the last two sections of the paper. In particular, the relation between the general model and discrete-time stochastic control models is brought out by applying the basic results to a linear stochastic control problem with state constraints.

Nonlinear programming. In "Monotone operators and augmented Lagrangians in nonlinear programming," [3], the Hestenes-Powell method of multipliers is modified to obtain a superior global convergence property under a stopping criterion that is easier to implement. The convergence results are obtained from the theory of the proximal point algorithm for solving $0 \in T(z)$ when T is a maximal monotone operator. An extension is made to an algorithm for solving variational inequalities with explicit constraint functions.

Computational methods of this type, using the theory of augmented Lagrangians, are now considered among the best for nonlinear programming problems. Much of the theory for such methods was developed under the predecessor of this grant.

Optimal control of dynamical systems. Papers [4] and [5] are in this area. In [4], "Duality in optimal control," an expository account is given of the way that the study of optimal control problems has been transformed by recent developments in convex analysis. Among the topics treated are existence of solutions, dual problems, generalized Hamiltonian equations, and state constraints versus impulse controls. This paper is about 60 pages long.

Paper [5], "Convex processes and Hamiltonian dynamical systems," describes in terms of models of optimal economic growth a number of results and open questions concerning optimal control problems over an infinite time interval. The main question in such problems is what kind of behavior is naturally optimal in a "self sustaining" sense, i.e. in a steady-state manner that could be prolonged indefinitely. The concepts that arise in this connection are

interesting for several basic reasons, especially as a description of limiting behavior in various situations, even though real problems never involve infinite time.

Convex analysis. A relatively minor contribution is made in "Higher derivatives of convex functions," [2]. This answers the question of what conditions are needed on a convex function in order to ensure that the conjugate function is m-times differentiable.

Are the standard optimality conditions in nonlinear programming "usually" satisfied? This is the question tackled by Jon Spingarn in his dissertation [6], an impressive piece of work. The question is important because it is not possible, as a practical matter in most applications, to check whether a given nonlinear programming problem (P) satisfies the "constraint qualifications" and strengthened forms of the second-order optimality conditions on which the analysis of many algorithms, etc., depends. It is often argued that it is all right to base results on the assumption of such conditions, because they hold in "typical" problems. But what does this assertion really mean?

Spingarn considers nonlinear programming problems $(P(w))$ that depend on a parameter vector w ranging over a set $W \subset R^k$. The components of w may represent costs, availabilities, demands, and other variables that are truly "variable" within the context of a particular model. Each parameterization of this sort gives a "class" of problems. Let us suppose w is actually a random vector governed by a probability measure on W that is expressed by a density function with respect to ordinary k-dimensional measure. We can say that a property is "typical" of the class $(P(w))$, $w \in W$, if the set of w for which it fails is a set of zero probability.

Spingarn's achievement has been to develop a strong and flexible theory of the parameterizations for which it is true that the constraint qualifications and second-order optimality conditions mentioned above "typically" hold. Besides the immediate implications already described, it is expected that Spingarn's

results will find application in areas like stochastic programming, which are concerned very directly with optimization problems that depend on random variables.

This, by the way, is a pioneering effort that may have many interesting consequences in optimization theory. No one has previously tried to provide a solid answer to this kind of question. Spingarn was supported for two years as a Research Assistant under this grant.

Articles of R. T. Rockafellar produced during this grant period.

[1] "The optimal recourse problem in discrete time: L^1-multipliers for inequality constraints," to appear in SIAM J. Control Opt. (Written with R. J. B. Wets).

[2] "Higher derivatives of conjugate convex functions," International J. Appl. Analysis 1 (1977), 41-43.

[3] "Monotone operators and augmented Lagrangian methods in nonlinear programming," to appear in Nonlinear Programming 3 (O. L. Mangasarian et al., editors), Academic Press, 1978.

[4] "Duality in optimal control," to appear in Mathematical Control Theory (W. A. Coppel, editor), Academic Press, 1978.

[5] "Convex processes and Hamiltonian dynamical systems," to appear.

Thesis of J. W. Spingarn completed during this grant period.

[6] Generic Conditions for Optimality in Constrained Minimization Problems, Ph.D. Dissertation, University of Washington, August, 1977.

Book manuscript of R. T. Rockafellar in preparation.

[7] Optimization in Networks, to be published by the Princeton University Press. The contents of the three chapters written in 1977 are as follows:

Chapter 7: Linear Optimization (66 pages)

- 7A. Linear Optimal Distribution Problem
- 7B. Examples of Optimization of Flows
- 7C. Linear Optimal Distribution Algorithm
- 7D. Simplex Algorithm for Flows
- 7E. Linear Optimal Differential Problem
- 7F. Examples of Optimization of Potentials
- 7G. Linear Optimal Differential Algorithm
- 7H. Simplex Algorithm for Potentials
- 7I. Elementary Problems and Duality
- 7J. Elementary Out-of-Kilter Algorithm
- 7K.. Exercises (31)